Received May 13, 1773.

XXXVI. *Properties of the Conic Sections; deduced by a compendious Method. Being a Work of the late* William Jones, *Esq; F. R. S. which he formerly communicated to Mr.* John Robertſon, *Libr. R. S. who now addreſſes it to the Reverend* Nevil Maſkelyne, *F. R. S. Aſtronomer Royal.*

SIR,

Read June 24, 1773.

YOU well know that the curves formed by the ſections of a cone, and therefore called CONIC SECTIONS, have, from the earlieſt ages of geometry, engaged the attention of mathematicians, on account of their extenſive utility in the ſolution of many problems, which were incapable of being conſtructed by any poſſible combination of right lines and circles, the magnitudes uſed in plane geometry. The properties of theſe curves are become far more intereſting within the two laſt centuries, ſince they have been found to be ſimilar to thoſe which are deſcribed by the motions of the cœleſtial bodies in the Solar ſyſtem.

Two

Two different methods have been taken by the writers who have treated of their properties; the one, and the more antient, is to deduce them from the properties of the cone itſelf; the other is to conſider the curves, as generated by the conſtant motion of two or more ſtrait lines moving in a given plane, by certain laws.

There are various methods of generating theſe curve lines *in plano*; one method will give ſome properties very eaſily; but others, with much trouble: while, by another mode of deſcription, ſome properties may be readily derived, which, by the former, were not ſo eaſily come at: ſo that it appears there may be a manner of deſcribing the curves ſimilar to the Conic Sections, by the motion of lines on a plane, which, in general, ſhall produce the moſt eſſential properties, with the greateſt facility.

That excellent mathematician, the late William Jones, Eſq; F. R. S. had drawn up ſome papers on the deſcription of theſe curves, or lines of the ſecond kind, very different from what he gave in his *Synopſis Palmariorum Matheſeos*, publiſhed in the year 1706; or from that of any other writer on this ſubject. A copy of theſe papers he was pleaſed to let me take about the year 1740. He had not finiſhed them as he intended; but, in their preſent ſtate, they appear of too much conſequence to be loſt; as, it is much to be feared, his own copy, together with many other valuable papers, are; and therefore, I am deſirous of preſerving them in the Philoſophical

Transactions, in the manner I at first transcribed them; although, I am aware, they might have been put into a form more pleasing to the generality of readers: I have indeed annexed larger diagrams than what accompanied the author's copy, in order to render the lines more distinct, as all the relations are to be represented in a single figure, of each kind.

Mr. Jones, having laid down a very simple method of describing these curves, seems to have been desirous of arriving at their properties in as expeditious a way as he could contrive; and therefore he has used the algebraic method, in general, of reducing his equations; and, on some occasions, has used the method of fluxions, to deduce some properties chiefly relating to the tangents; and, by a judicious use of these, he has very much abridged the steps which otherwise he must have taken, to have deduced the very great variety of relations he has obtained: these he intended to have arranged in tables, from whence an equation expressing the relation between any three or more lines of the Conic Sections, might be taken out as readily as a logarithm out of their tables; this he has only partly executed; but it may easily be continued by those who are desirous to have it done, and are sufficiently acquainted with what follows.

From the House of the Royal Society,
April 29, 1773.

THE

The description of Lines of the second kind.

LET the right lines AD, AQ, be drawn on a plane, at any inclination the one with the other. See Plate xiv. Fig. 1, 2, 3.

In AD, AQ, take A*a*, AM, of any given magnitude, and draw MN parallel to AD,

On the points A, *a*, let two rulers AP, *a*P, revolve, and cut MN, AQ, in N and Q, ſo that AQ be every-where equal to MN.

Then ſhall the interſection P of the rulers deſcribe lines of the ſecond kind, or curves of the firſt kind.

Where the right-line A*a*, is the firſt, or tranſverſe diameter.

The point C, biſecting the diameter A*a*, is the center.

The right-line PD, drawn parallel to AQ, is the ordinate to the diameter A*a*.

The part AD, or CD, of the diameter, is the abſciſs, when reckoned to begin from A to C, or from C to A.

The right line B*b* drawn from the center C parallel to the ordinate PD, and terminated in the curve, is called the ſecond, or conjugate diameter.

Thoſe diameters to which the ordinates are perpendicular, are called the axes.

And AM is the parameter to the diameter A*a*.

THE

THE PROPERTIES OF LINES OF THE SECOND KIND.

1. Put $Aa=d=2AC=2t$; $Bb=\delta=2BC=2c$; $AM=2p$; $PD=y$; $CD=x$; $AD=u$. Then $\overline{PD}^2=\frac{p}{t}\times ADa$.

Or $yy=\frac{p}{t}\times u\times\overline{d\mp u}=2pu\mp\frac{p}{t}uu=\frac{p}{t}\times\overline{\pm tt\mp xx}=\pm pt\mp\frac{p}{t}x^2$.

For $PD=\frac{AM\times AD}{MN}=\frac{AQ\times Da}{Aa}$ (by sim. $\triangle$s). Th. $\overline{PD}^2=\frac{AM}{Aa}\times ADa$.

2. Consequently $\frac{t}{p}yy=\pm tt\mp xx=du\mp uu=\pm\frac{1}{4}dd\mp xx$.

3. Hence $\frac{t}{p}y\dot{y}=\mp x\dot{x}=\overline{t\mp u}\times\dot{u}=x\dot{u}$.

4. And $pt=cc$, or $2pd=\delta\delta$; for when $y=c=\frac{1}{2}\delta$, then $x=0$.

5. Therefore $\frac{\overline{PD}^2}{ADa}=\frac{yy}{\pm tt\mp xx}=\frac{p}{t}=\frac{2p}{d}=\frac{2pd}{dd}=\frac{pt}{tt}=\frac{\delta\delta}{dd}=\frac{cc}{tt}=\frac{\overline{BC}^2}{\overline{AC}^2}$.

6. The curve line whose property is $yy+\frac{p}{t}uu-2pu=0$,
(where the abscissa begins at the curve),
Or $yy+\frac{cc}{tt}xx-cc=0$, (where it begins at the center),
is called an Ellipsis. This curve returns into itself. For when $x=0$, then $y=c$; and when $y=0$, then $x=t$. Which can happen but two ways.

7. The curve line whose property is $yy-\frac{p}{t}uu-2pu=0$,

Or $yy-\frac{cc}{tt}xx+cc=0$, is called an Hyperbola. This curve spreads out infinitely. For y increases as x increases; and has four legs tending contrary ways: for xx, or yy, may be produced as well from $-x$, or $-y$; as from $+x$, or $+y$.

8. If the point a, is supposed to be at an infinite distance from A, so that a ruler aP moves in a parallel position to AD; then is $yy=2pu$, or $yy-2pu=0$, the property of the curve described, and is called a Parabola. This curve spreads out infinitely; for y increases as u increases.

9. Let

9. Let Aa, Pp, be any two first diameters; Bb, Qq, their second diameters. PLATE XIV. Fig. 4.

Draw the ordinates PD, QE, to the diameter Aa, and the ordinate Pd, to the diameter B b.

Let PT be a tangent, and PM be perpendicular, to the curve, in P; PT cutting Aa, Bb, produced, in T, t; and PM, in M, m.

Put the subtangent DT $= s$, $dt = \sigma$.

Let AT $= r$, PM $= \pi$, CE $= w$, CT $= q = x \pm s = t \pm r$.

Put s, s', = sine and cosine of the angle MPD, or angle PmB.

R = tabular radius.

Then $CDT = xs = \overline{t \mp u} \times s = \frac{t}{p} yy = \frac{tt}{cc} yy = \pm tt \mp xx = u \times \overline{d \mp u} = ADa$.

For $\dot{u} \times \overline{t \mp u} = \left(\frac{t}{p} y\dot{y} =\right) \alpha y \dot{y}$. Therefore $\left(\frac{\dot{u}}{\dot{y}} = \frac{\alpha y}{t \mp u} =\right) \frac{\alpha y}{x} = \frac{s}{y}$ (by sim. $\triangle$s).

And $Cdt = y\sigma = \frac{p}{t} xx = \frac{cc}{tt} xx = \pm cc \mp yy = Bdb$. For $\left(\frac{\dot{u}}{\dot{y}} =\right) \frac{\alpha y}{x} = \frac{x}{\sigma}$.

10. Hence $\overline{AC}^2 = tt = x \times \overline{x \pm s} = x \times \overline{t \pm r} = xq = DCT$.

11. And $\overline{BC}^2 = cc = y \times \overline{y + \sigma} = y \times Ct = dCt$.

12. Consequently $\frac{\overline{Pd}^2}{Bdb} = \frac{xx}{\pm cc \mp yy} = \frac{tt}{cc} = \frac{\overline{AC}^2}{\overline{BC}^2}$.

13. Also $\overline{CE}^2 = ww = \left(\overline{DT}^2 \times \frac{\overline{QE}^2}{\overline{PD}^2} = \overline{DT}^2 \times \frac{AEa}{ADa} =\right) ss \times \frac{tt - ww}{sx} = sx = \frac{tt}{cc} yy = ADa = CDT$.

14. Therefore $AEa = \left(\frac{\overline{CE}^2}{\overline{DT}^2} \times ADa = \frac{sx}{ss} \times sx =\right) xx = \overline{CD}^2$.

15. And $\overline{QE}^2 = \left(\frac{\overline{PD}^2}{ADa} \times AEa =\right) \frac{cc}{tt} xx = \frac{p}{t} xx$.

In

In the general schemes. PLATE XV. and Fig. 5. PLATE XIV.

16. Let Aa, Bb, be the longest and shortest axes.
Draw $C\varrho$ perpendicular to the tangent PT, cutting it in ϱ.
Put $CP = T$; $CQ = C$; $C\varrho = \varrho$.

Then $C\varrho = \varrho = \left(\frac{Ct \times CE}{CQ} = \frac{cc}{yy} \times \frac{ty}{cC} =\right) \frac{tc}{C} = \frac{AC \times CB}{CQ}$.

Hence the parallelogram, under the two axes, is equal to the parallelogram under any two diameters.

17. Draw the tangents AN, an, to any vertices A, a, meeting any diameters Pp, Qq, produced in V, U, and v, u, and the tangent PT in N, n.

Then $AU = \left(\frac{CA \times EQ}{CE} =\right) \frac{px}{p}$. And $AV = \left(\frac{CA \times PD}{CD} =\right) \frac{ty}{x}$.

18. Also $CU = \left(\frac{AU \times CQ}{EQ} =\right) \frac{cC}{y}$. And $CV = \left(\frac{CP \times CA}{CD} =\right) \frac{tT}{x}$.

19. Hence $PV = (CV \backsim CP =)\ T \times \frac{t \backsim x}{x}$. And $pV = T \times \frac{t+x}{x}$.

Also $QU = (CU \backsim CQ =)\ C \times \frac{c \backsim y}{y}$. And $qU = C \times \frac{c+y}{y}$.

20. When Aa and Bb are the longest and shortest axes; and when $y = p$,
Then $xx = tt \mp \frac{t}{p} yy$ will become $tt \mp pt = tt \mp cc$, which call ff.
And $CD = x$, will become $CF = Cf = f$.
The points F, f, are called the Focii.

21. Hence $AF = af = \pm t \mp f$; $Af = aF = t + f$.

22. Also $\overline{CF}^2 = \overline{Cf}^2 = ff = \pm tt \mp cc = \pm tt \mp pt$.
And in the ellipsis, $\overline{AC}^2 = \left(\overline{BC}^2 + \overline{CF}^2 =\right) \overline{BF}^2$.
in the hyperbola, $\overline{CF}^2 = \left(\overline{AC}^2 + \overline{BC}^2 =\right) \overline{BA}^2$.
Hence, a circle described from B, with the distance AC in the ellipsis, or from C, with the distance AB in the hyperbola, will cut the axis Aa in the focii F, f.

23. Draw

23. Draw FP, fP, from the focii F, f, to any point P of the curve; and draw the conjugate diameters Pp, Qq.

Put $PF = z$; $Pf = v$; $PC = T$; $QC = C$; $\frac{f}{t} = \varphi$; $\frac{f}{c} = \gamma$. Then

$$\overline{Pf}^2 = vv = yy + xx + 2xf + ff = tt + 2fx + \frac{ff}{tt}xx = TT + ff + 2fx.$$

$$\overline{PF}^2 = zz = yy + xx - 2xf + ff = tt - 2fx + \frac{ff}{tt}xx = TT + ff - 2fx.$$

For $yy = (\frac{cc}{tt} \times \overline{tt \backsim xx} =) \frac{tt \backsim ff}{tt} \times \overline{tt \backsim xx}.$

24. $$Pf = v = \left(\sqrt{tt + 2fx + \frac{ff}{tt}xx} =\right) t + \frac{f}{t}x = t + \varphi x = \frac{tt + fx}{t}.$$

$$PF = z = \left(\sqrt{tt - 2fx + \frac{ff}{tt}xx} =\right) t - \frac{f}{t}x = t - \varphi x = \frac{tt - fx}{t}.$$

25. $PF \pm Pf = z \pm v = 2t = Aa.$

26. $Pf^2 + \overline{PF}^2 = vv + zz = 2yy + 2xx + 2ff = 2tt + 2\varphi\varphi xx = 2TT + 2ff.$

27. $\overline{Pf}^2 - \overline{PF}^2 = vv - zz = 4fx = \overline{v + z} \times \overline{v - z} = 2t \times \overline{v \mp z}.$

28. $$FPf = zv = z \times \overline{2t \mp z} = 2tz \mp zz = tt \backsim \varphi\varphi xx = \overline{tt \backsim xx} \times \frac{cc}{tt}xx$$

$$= cc \backsim \frac{ff}{cc}yy = \frac{tt}{cc}yy + \frac{cc}{tt}xx = \overline{CE}^2 + \overline{EQ}^2$$

$$= \overline{CQ}^2 = CC = T \times P.$$

29. Let $m = t - z = v - t = \frac{fx}{t} = \varphi x = \pm \frac{1}{2}v \mp \frac{1}{2}z$

$$= \frac{fx}{\sqrt{cc + ff}} = \frac{x}{t}\sqrt{tt \mp cc} = \sqrt{tt \mp zv}$$

$$= \sqrt{tt \mp CC} = \sqrt{ff + cc - zv}.$$

30. Hence $\dot{z} = -\dot{v} = \varphi\dot{x} = -\frac{f}{t}\dot{x} \pm \frac{fy}{px}\dot{y}.$

And $\dot{v} = -\dot{z} = \varphi\dot{x} = \frac{f}{t}\dot{x} = -\frac{fy}{px}\dot{y}.$

31. $AC = t = \frac{1}{2}z \pm \frac{1}{2}v = \frac{cc}{p} = \sqrt{cc \pm ff} = \frac{fx}{m} = \frac{cx}{\sqrt{cc \backsim yy}}$

$= \frac{c}{y}\sqrt{sx} = \sqrt{x \times x \backsim s} = \frac{y^2 + \sqrt{y^4 + 4p^2x^2}}{2p} = \frac{yy}{2p} + \sqrt{\frac{y^4}{4p^2} + xx}.$

32. $\overline{CD}^2 = \overline{Pd}^2 = xx = \left.\frac{ttyy}{ccs}\right|^2 = \left.\frac{tyy}{ps}\right|^2 = \pm tt \mp \frac{t}{p}yy = \frac{tt}{cc} \times \overline{cc \mp yy}$

$= \frac{ff+cc}{cc} \times \overline{cc \mp yy} = tt \mp \frac{tt}{cc}yy = \frac{mm}{\varphi\varphi} = \frac{tt}{ff}mm$

$= \frac{cc+ff}{ff}mm = \frac{tt}{ff} \times \overline{tt \mp zv} = \frac{tt}{ff} \times \overline{tt \mp CC} = \left.\frac{tt}{q}\right|^2$

$= \frac{tt}{cc} \times zv - \frac{tt}{cc}yy = \frac{t}{p} \times zv - \frac{t}{p}yy = \frac{tt}{ff} \times \left.\overline{t - z}\right|^2$

$= \frac{tt}{ff} \times \left.\overline{v - t}\right|^2 = \left.\frac{tt \mp tz}{f}\right|^2 = \left.\overline{t \mp u}\right|^2 = \left.\overline{\sqrt{tt + \frac{1}{4}ss} - \frac{1}{2}s}\right|^2$

$= \frac{tt}{ff} \times \overline{tt \mp \frac{tt}{cc}\pi\pi} = \frac{t^4}{c^4} \times \frac{SS}{RR}\pi\pi.$

33. And $\dot{x} = -\frac{ty\dot{y}}{px} = -\frac{tty\dot{y}}{ccx} = -\frac{tt\dot{y}}{qq} = -\frac{t\dot{z}}{f} = \frac{t\dot{v}}{f}.$

34. $\overline{PD}^2 = \overline{Cd}^2 = yy = \frac{p}{t}sx = \frac{cc}{tt}sx = \frac{cc}{tt} \times \overline{\pm tt \mp xx} = \pm cc \mp \frac{cc}{tt}xx$

$= \pm pt \mp \frac{p}{t}xx = \frac{2cc}{t}u \mp \frac{cc}{tt}uu = \frac{\pm tt \mp ff}{tt} \times \overline{\pm tt \mp xx}$

$= \frac{cc}{tt} \times zv - \frac{cc}{tt}xx = \frac{cc}{tt} \times CC - \frac{cc}{tt}xx = \frac{cc}{ff} \times \overline{zv - cc}$

$= \frac{cc}{ff}nn = \frac{pt}{ff}nn$ (putting $nn = zv - cc$)

$= \frac{tt\pi\pi - c^4}{ff} = cc \mp \frac{tt}{cc} \times \frac{SS}{RR}\pi\pi = \frac{S'S'}{RR}\pi\pi.$

35. And $\dot{y} = -\frac{px\dot{x}}{ty} = -\frac{ccx\dot{x}}{tty} = \frac{px\dot{z}}{fy} = -\frac{px\dot{v}}{fy}.$

36. Also $\frac{ff}{cc}yy = zv - cc = \frac{ff}{tt} \times \overline{tt \mp xx} = ff \mp \frac{ff}{tt}xx$

$= nn = ff \mp mm = CC - cc.$

37. $\overline{PC}^2 = TT = xx + yy = xx \pm cc \mp \frac{cc}{tt}xx = tt - ff + \frac{ff}{tt}xx = cc + \varphi\varphi xx$
$= cc + tt \backsim zv = cc + mm = cc + tt \backsim CC$
$= tp + \varphi\varphi xx = 2cc + ff \backsim zv = 2pt + ff \backsim zv.$

38. And $\dot{T} = \frac{\varphi\varphi x\dot{x}}{T} = \frac{\varphi\varphi x\dot{x}}{\sqrt{tp+\varphi\varphi xx}} = \left(\frac{\varphi\varphi x}{T} \times -\frac{ty\dot{y}}{px} =\right)\frac{ffy\dot{y}}{ptT} = -\frac{ffy\dot{y}}{ccT}.$

39 Also $\overline{CQ}^2 + \overline{CP}^2 = (cc + tt =)\ \overline{CB}^2 + \overline{CA}^2.$

40. $CT = q = \frac{tt}{x} = \left(tt \times \frac{f}{tm} =\right)\frac{ft}{m} = \frac{ft}{t \backsim z} = \frac{ft}{\sqrt{tt \backsim zv}}$

$Ct = \frac{cc}{y} = \frac{cf}{n} = \frac{cf}{\sqrt{zv - cc}}.$

41. And $\dot{q} = -\frac{q\dot{x}}{x} = -\frac{qq\dot{x}}{tt} = \frac{qqy\dot{y}}{ccx} = \frac{qqy\dot{y}}{ptx} = -\frac{tt\dot{x}}{xx}$ (for $tt = qx$).

42. $AT = (\pm CT \mp CA =) \pm\frac{tt}{x} \mp t = \frac{t}{x} \times \pm\overline{t \mp x} = \pm\frac{ft}{m} \mp t = \pm\frac{ft}{t \mp z} \mp t.$

$aT = (CT + Ca =)\ \frac{tt}{x} + t = \frac{t}{x} \times \overline{t+x} = \frac{ft}{m} + t = \frac{ft}{t \backsim z} + t.$

43. $FT = \left(\pm CT \mp CF = \pm\frac{ft}{m} \mp f = \frac{\pm ft \mp fm}{m}\right) = \frac{fz}{t \backsim z} = \frac{fz}{m} = \frac{tz}{x}.$

$fT = \left(CT + Cf = \frac{ft}{m} + f = \frac{ft+fm}{m} =\right)\frac{fv}{v \backsim t} = \frac{fv}{m} = \frac{tv}{x}.$

On Aa describe a circle, draw the tangent TP', and draw CP'.
Continue DP to P', and at right angles to TP', draw FR',

Then $FR' = \left(\frac{FT}{CT} \times CP' = \frac{tz}{x} \times t \times \frac{x}{tt} =\right) z = FP.$

$fr' = \left(\frac{fT}{CT} \times CP' = \frac{tv}{x} \times t \times \frac{x}{tt} =\right) v = fP.$

44. DT

44. $\text{DT} = s = \frac{tyy}{px} = \frac{ttyy}{ccx} = \frac{\pm tt \mp xx}{x} = \frac{tnn}{fm}$.

$dt = \sigma = \frac{pxx}{ty} = \frac{ccxx}{tty} = \frac{cc \mp yy}{y} = \frac{cmm}{fn}$.

45. $\text{AN} = \left(\frac{\text{C}t}{\text{CT}} \times \text{AT} =\right) \frac{cm}{nx} \times \overline{t - x} = \frac{c}{n} \times \overline{f - m}$.

$an = \left(\frac{\text{C}t}{\text{CT}} \times a\text{T} =\right) \frac{cm}{nx} \times \overline{t + x}$.

46. $\text{VN} = \left(\frac{\text{PV} \times \text{C}t}{\text{PC}} = \frac{t - x}{\text{T}} \times \frac{\text{T}}{x} \times \frac{fc}{n} =\right) \frac{fc}{nx} \times \overline{t - x} = \frac{fc}{nm} \times \overline{f - m}$

$= \frac{fc}{\sqrt{zv - cc} \times \sqrt{tt - zv}} \times \overline{f - \sqrt{tt - zv}}$

$\text{UN} = \text{C}t = \frac{cc}{y}$.

47. $\text{AD} = \left(\pm \text{AC} \mp \text{CD} = \pm t \mp \frac{tm}{f} =\right) \frac{t}{f} \times \overline{\pm f \mp m}$

$a\text{D} = \left(\text{AC} + \text{CD} = t + \frac{tm}{f} =\right) \frac{t}{f} \times \overline{f + m}$.

48. Produce PF, pf, to meet the curve in Π, π;

Draw $\Pi\Delta$ perpendicular to Aa.

Put $\text{F}\Pi = z'$, $f\pi = v'$, $\text{F}\Delta = x'$.

Now, $\text{FD} = (\pm \text{CF} \mp \text{CD} =) \pm f \mp x = \pm f \mp \frac{tm}{f} = \frac{tz - cc}{f}$

$f\text{D} = f + \frac{tm}{f} = \frac{tv - cc}{f}$.

Then $\text{F}\Delta = x' = (\text{C}\Delta - \text{CF} =) \frac{tm}{f} - f = \left(\frac{t}{f} \times \overline{t - z'} - f =\right) \frac{cc - tz}{f}$

$f\Delta = f + \frac{tm}{f} = \frac{tv' - cc}{f}$.

49. From

49. From F, f, draw FR, fr, perpendicular to the tangent PT, aud cutting it in R, r.

$$\text{FR} = \lambda = \frac{\text{FT} \times \text{C}\varrho}{\text{CT}} = \frac{cz}{\text{C}} = \frac{cz}{\sqrt{zv}} = \frac{cz}{\sqrt{2tz - zz}} = c\sqrt{\frac{z}{v}}$$

$$= c\sqrt{\frac{t - \phi x}{t + \phi x}} = \sqrt{\frac{ptz}{2t - z}} = \frac{z\varrho}{t}$$

$$fr = \lambda' = \frac{f\text{T} \times \text{C}\varrho}{\text{CT}} = \frac{cv}{\text{C}} = \frac{cv}{\sqrt{zv}} \; \cdots\cdots = c\sqrt{\frac{z}{v}}$$

$$= e\sqrt{\frac{t + \phi x}{t - \phi x}} = \sqrt{\frac{ptv}{2t - v}}.$$

$$\dot{\lambda} = \frac{tc\dot{z}}{v\sqrt{zv}}. \quad \text{For } \lambda^2 = \frac{c^2 z}{v}. \quad \text{Th. } 2\lambda\dot{\lambda} = \left(\frac{c^2 v\dot{z} - c^2 z\dot{v}}{vv} = \right) \frac{2c^2 t\dot{z}}{vv}.$$

50. $$\text{TR} = \left(\frac{\text{AT} \times \text{RF}}{\text{AN}} = \frac{tnz}{m\text{C}} = \right) \frac{ttnz}{f\text{C}x} = \frac{ttn}{fx}\sqrt{\frac{z}{v}} = \frac{tn}{m}\sqrt{\frac{z}{v}}.$$

$$\text{T}r = \frac{a\text{T} \times rf}{an} = \frac{tn}{m}\sqrt{\frac{v}{z}}.$$

51. Draw PM perpendicular to the tangent PT, meeting the axes Aa, Bb, in M, m.

$$\text{DM} = \left(\frac{\text{PD} \times \text{PD}}{\text{TD}} = \right) \frac{ccm}{ft} = \frac{cc}{ft}\sqrt{tt \pm zv} = \frac{cc}{tt}x = \frac{p}{t}x$$

$$= \frac{\text{S}}{\text{R}}\pi = p \mp \frac{p}{t}u = \frac{cc}{tf}\sqrt{tt \mp \frac{tt}{cc}\pi\pi}.$$

$$dm = \left(\frac{\text{PD} \times \text{P}d}{\text{DM}} = \right) \frac{ttn}{fc} = \frac{tt}{cc}\sqrt{zv - cc} = \frac{tt}{cc}y = \frac{t}{p}y.$$

52. $$\text{CM} = \left(\text{CD} \mp \text{DM} = x \mp \frac{cc}{tt}x = \right) \frac{tt \mp cc}{tt}x = \frac{ff}{tt}x = \frac{f}{t}m$$

$$= \frac{f}{t}\sqrt{tt \mp zv} = \frac{f}{t} \times \overline{t \mp z} = \frac{f}{t} \times \sqrt{tt \mp \frac{tt}{cc}\pi\pi}$$

$$\text{C}m = \left(dm \mp \text{C}d = \frac{tt}{cc}y \mp y = \right) \frac{tt \mp cc}{cc}y = \frac{ff}{cc}y = \frac{f}{c}n = \frac{f}{c}\sqrt{zv - cc}$$

53. $$\text{FM} = \left(\text{CF} \backsim \text{CM} = f \backsim \frac{fm}{t} = \right) \frac{f}{t} \times \overline{t \mp m} = \frac{fz}{t} = \pm f \mp \frac{ff}{tt}x.$$

$$f\text{M} = \left(f\text{C} + \text{CM} = f + \frac{fm}{t} = \right) \frac{f}{t} \times \overline{t + m} = \frac{fv}{t} = f \pm \frac{ff}{tt}x.$$

54. $\overline{Fm}^2 = \overline{fm}^2 = (\overline{cf}^2 + \overline{cm}^2 =)\ ff + \frac{ffnn}{cc} = \frac{ffzv}{cc} = \frac{ff}{cc} \times CC.$

55. $AM = (AC \backsim CM =)\ t \backsim \frac{fm}{t} = \frac{tt \backsim fm}{t} = t \backsim \frac{ff}{tt}x.$

$aM = (aC + CM =)\ t + \frac{fm}{t} = \frac{tt + fm}{t} = t + \frac{ff}{tt}x.$

56. $TM = (TF + FM = \frac{fz}{m} + \frac{fz}{t} =)\ \frac{fzv}{tm} = \frac{zv}{x} = \frac{CC}{x}$

$tm = (Ct + Cm = \frac{fc}{n} + \frac{fn}{c} =)\ \frac{fzv}{cn} = \frac{zv}{y} = \frac{CC}{y}.$

57. $PM = \pi = \left(\frac{FR \times TM}{FT} =\right)\ \frac{c}{t}\sqrt{zv} = \frac{c}{t}C = \frac{p}{c}C = \frac{c}{t}\sqrt{tt \backsim \varphi^2 x^2}$

$= \frac{c}{t}\sqrt{\overline{t+\varphi x} \times \overline{t-\varphi x}} = \frac{cc}{tt} \times \frac{R}{S}x = \frac{cR}{tS}\sqrt{ce \mp yy}$

$= \frac{c}{tt}\sqrt{t^4 - ffxx} = \frac{c}{t}\sqrt{cc + \gamma\gamma yy} = \frac{1}{t}\sqrt{c^4 + ffyy}$

$= \frac{c}{t}\sqrt{2tz \mp zz} = \sqrt{2pz \mp \frac{p}{t}zz}.$

$pm = \pi' = \left(\frac{PM \times Pd}{DM} =\right)\ \frac{t}{c}\sqrt{zv} = \frac{t}{c}C = \frac{c}{p}C = \frac{t}{c}\sqrt{tt \backsim \varphi\varphi xx}$

$= \frac{t}{c}\sqrt{\overline{t+\varphi x} \times \overline{t-qx}} = \frac{1}{c}\sqrt{t^4 \backsim ffxx} = \frac{t}{c}\sqrt{cc + \gamma\gamma yy}$

$= \frac{t}{cc}\sqrt{c^4 + ffyy} = \frac{t}{c}\sqrt{2tz \mp zz} = \sqrt{\frac{2tt}{p}z \mp \frac{t}{p}zz}.$

58. $Mm = (Pm \mp PM = \frac{t}{c}C \mp \frac{c}{t}C =)\ \frac{tt \mp cc}{ct}C = \frac{ff}{ct}\sqrt{zv}.$

$= \frac{ff}{ct}\sqrt{tt \backsim \varphi\varphi xx}.$

59. $PT = \tau = \left(\frac{PD \times PM}{DM} =\right)\ \frac{n}{m}C = \sqrt{\frac{zv - cc}{tt - zv} \times zv} = \frac{tn}{fx}C.$

$= \frac{ty}{cx}C = \frac{ty}{cx}\sqrt{tt \backsim \varphi\varphi xx} = \frac{\sqrt{\overline{zv - cc} \times zv}}{t \mp x}.$

$Pt = \tau' = \left(\frac{PT \times Pd}{DT} =\right)\ \frac{m}{n}C = \sqrt{\frac{tt - zv}{zv - cc} \times vz} = \frac{fx}{tn}C$

$= \frac{cx}{ty}C = \frac{cx}{ty}\sqrt{tt - \varphi\varphi xx}.$

60. TN

60. $\text{TN} = \left(\frac{\text{PT}\times\text{TA}}{\text{DT}} = \frac{n\text{C}}{m}\times\frac{t}{m}\times\overline{f\backsim m}\times\frac{fm}{tnn} = \right)\frac{f\text{C}}{mn}\times\overline{f\backsim m} = \frac{c\text{C}}{xy}\times\overline{t\backsim x}$

$\text{T}n = \left(\frac{\text{PT}\times\text{T}a}{\text{DT}} = \frac{n\text{C}}{m}\times\frac{t}{m}\times\overline{f+m}\times\frac{fm}{tnn} = \right)\frac{f\text{C}}{mn}\times\overline{f+m} = \frac{c\text{C}}{xy}\times\overline{t+x}$

61. $\text{PN} = \left(\frac{\text{AD}\times\text{PT}}{\text{DT}} = \frac{t}{f}\times\overline{f\backsim m}\times\frac{n\text{C}}{m}\times\frac{fm}{tnn} = \right)\frac{\text{C}}{n}\times\overline{f\backsim m} = \frac{c\text{C}}{ty}\times\overline{t\backsim x}$

$\text{P}n = \left(\frac{a\text{D}\times\text{PT}}{\text{DT}} = \frac{t}{f}\times\overline{f+m}\times\frac{n\text{C}}{m}\times\frac{fm}{tnn} = \right)\frac{\text{C}}{n}\times\overline{f+m} = \frac{c\text{C}}{ty}\times\overline{t+x}$

62. $\text{PR} = \left(\frac{\text{PT}\times\text{FM}}{\text{TM}} = \right)\frac{nz}{\text{C}} = n\sqrt{\frac{z}{v}} = \sqrt{\frac{z}{v}\times\overline{zv-cc}} = \frac{f}{c}y\times\sqrt{\frac{z}{v}}$

$$= \frac{f}{c}y\times\sqrt{\frac{t-\varphi x}{t+\varphi x}} = \frac{f}{cc}\times\lambda y$$

$\text{P}r = \left(\frac{\text{PT}\times f\text{M}a}{\text{TM}} = \right)\frac{nv}{\text{C}} = n\sqrt{\frac{v}{z}} = \sqrt{\frac{v}{z}\times\overline{zv-cc}} = \frac{f}{c}\times y\sqrt{\frac{v}{z}} = \frac{f}{c}y\times\sqrt{\frac{t+\varphi x}{t-\varphi x}}$

63. $\text{T}\varrho = \left(\frac{\text{PT}\times\text{TC}}{\text{TM}} = \right)\frac{ntt}{m\text{C}} = \frac{tt}{t-z}\sqrt{\frac{zv-cc}{vz}} = \frac{fytt}{mc\text{C}} = \frac{yt^3}{xc\text{C}}$

64. $\text{P}\varrho = \left(\text{T}\varrho \mp \text{TP} = \frac{ntt}{m\text{C}} \mp \frac{n\text{C}}{m}\times\right)\frac{n}{m\text{C}}\times tt\mp\text{CC} = \frac{n}{m\text{C}}\times mm = \frac{nm}{\text{C}}.$

65. $\text{T}t = \left(\frac{\text{PT}\times\text{C}t}{\text{PD}} = \frac{n\text{C}}{m}\times\frac{cf}{n}\times\frac{f}{cn} = \right)\frac{ff\text{C}}{mn} = ff\sqrt{\frac{zv}{\overline{tt-zv}\times\overline{zv-cc}}}$

$$= \frac{tc}{xy}\text{C} = \frac{tc}{xy}\sqrt{tt-\frac{ff}{tt}xx}$$

66. $\text{N}n = \left(\frac{\text{PT}\times\text{A}a}{\text{DT}} = \right)\frac{2f\text{C}}{cn} = \frac{2c}{y}\text{C} = \frac{2c}{y}\sqrt{zv} = 2f\sqrt{\frac{zv}{zv-cc}}$

From C and F, draw Cγ and Fε, perpendicular to Pf

Put Pε $= b$; Σ = *co*-fine of the $\angle$ AfP; R = tabular radius.

Then $f\gamma = \left(\frac{\text{C}f\times f\text{D}}{\text{P}f} = \frac{tv-cc}{f} = \right)\frac{tv-cc}{v} = \left(\frac{\Sigma}{\text{R}}\times\text{C}f\right)\frac{\Sigma f}{\text{R}}$

And $\text{P}f = v = \frac{cc}{t-\frac{\Sigma}{\text{R}}f} = \frac{tt-ff}{t-\frac{\Sigma}{\text{R}f}}$

$$Ps = h = \left(\frac{\pm zz + vv \mp 4ff}{2v} = \frac{\pm TT \mp ff}{v} = \frac{\pm 2pt \mp vz}{v} = \right)$$
$$= \pm \frac{2pt}{v} \mp z = \pm \frac{z+v}{v} p \mp z.$$

Hence $p = \frac{\pm h \mp z}{z+v} \times v$; $v = \frac{pz}{\mp h \pm z - p}$; $z = \frac{p-h}{v-p} \times v$

Therefore p is leſs than, equal to, or greater than $\pm h \mp z$, in the Ellipſis, Parabola, or Hyperbola.

67. Draw $m\mu$, making the $\angle\, Pm\mu = \angle\, PCM$, and cutting PC in μ.

Then $P\mu = \left(\frac{PM \times Pm}{PC} = \right) \frac{CC}{T} = \frac{zv}{\sqrt{tt \pm cc \mp zv}} = \frac{tt - \varphi\varphi xx}{\sqrt{cc + \varphi\varphi xx}}$

$= P = \frac{1}{2}$ parameter to PC.

68. Let $F\Phi$ be an ordinate to the axis Aa. at the focus F. and ΦG a tangent to the curve in Φ, meeting Aa, BC, AN, an, in G, g, S, s. Then

$$F\Phi = \sqrt{\frac{cc}{tt} \times AFa} = \sqrt{\frac{cc}{tt} \times cc} = \Big) \frac{cc}{t} = \frac{pt}{t} = p$$

$= \frac{1}{2}$ parameter to Aa.

69. $CG = \frac{tt}{f} = \frac{ff \pm cc}{f} = \frac{tt}{\sqrt{tt - cc}} = \frac{tt}{\sqrt{t \times t - p}}$

70. $FG = (CG \backsim CF = \frac{tt}{f} \backsim f = \frac{tt \backsim ff}{f} = \frac{cc}{f} = \frac{pt}{f}$.

71. Draw PH parallel to Aa; and GH perpendicular to Aa, meeting, PH, PT, in H, h; then

$$PH = DG = (CG \backsim DC =) \frac{tt}{f} \backsim x = \frac{t}{f} z = \frac{t}{f} \times PF.$$

72. $\text{TG} = (\text{CT} \backsim \text{CG} = \frac{ft}{m} \backsim \frac{tt}{f} = \frac{t}{m} \times \overline{f \backsim x} =) \frac{t}{t-z} \times \frac{\pm tz \mp cc}{f} = \frac{tt}{fx} \times \overline{f \backsim x}.$

73. $\text{G}h = (\frac{\text{PD} \times \text{TG}}{\text{DT}} = \frac{cn}{f} \times \frac{t}{m} \times \overline{f \backsim x} \times \frac{fm}{tnn} = \frac{\text{C}}{n} \times \overline{f \backsim x} =)$

$$\frac{cc}{fy} \times \overline{f \backsim x} = \frac{c}{\sqrt{zv - cc}} \times \frac{tz - cc.}{f}$$

74. $\text{P}h = (\frac{\text{TP} \times \text{DG}}{\text{DT}} =) \frac{z}{n}\text{C} = \frac{cz}{fy}\text{C} = z \times \sqrt{\frac{zv}{zv - cc}} = \frac{cz}{fy} \sqrt{zv}.$

75. $\text{R}h = (\text{P}h - \text{PR} =) \frac{z\text{C}}{n} - \frac{zn}{\text{C}} = \frac{z}{n\text{C}} \times \overline{\text{CC} - nn} =)$

$$\frac{zcc}{n\text{C}} = \frac{zcc}{\sqrt{zv - cc + zv}} = \frac{zc^3}{fy\text{C}}$$

76. $\text{F}h = (\sqrt{\overline{\text{FR}}^2 + \overline{\text{R}h}^2} =) \frac{cz}{n} = \frac{cz}{\sqrt{zv - cc}} = \frac{ccz}{fy} = \frac{ptz.}{fy}$

77. Let $\text{D}\Sigma$ be any Ordinate to the axe $\text{A}a$, cutting the curve in Σ, and the focal tangent ΦG in σ;

Then $\text{D}\sigma = (\frac{\text{F}\Phi + \text{DG}}{\text{FG}} = \frac{cc}{t} \times \frac{tz}{f} \times \frac{f}{cc} =) z = \text{FP} = \text{F}\Sigma.$

78. Therefore $\text{AS} = \text{AF}$; $as = a\text{F}$; $\text{C}g = \text{CA}$; by sim. $\triangle$s.

79. $\overline{\text{DF}}^2 = (\overline{\text{D}\sigma}^2 - \overline{\text{D}\Sigma}^2 = \overline{\text{D}\sigma + \text{D}\Sigma} \times \overline{\text{D}\sigma - \text{D}\Sigma} =) \text{P}\sigma \times \sigma\Sigma.$

79. Let the tangents PN, pL, to the opposite vertices P, p, cut the tangents AN, an, to the opposite vertices A, a, in N, n, L, l.
Then Pn = pL; an = AL; PN = pl; AN al.
For the Trapezia's PCan, pCAL, are similar and equal;
And so are the Trapezia's PCAN, pCal.

81. $PF \times Mf = \left(\frac{f}{t} \times vz = \right) Pf \times MF = \frac{f}{t} \times FP\,f.$

82. $CEQ = (xy =)\ CDP = d\,CD = DM \times dm.$

83. $AN \times aD = \frac{ctn}{f} = an \times AD = AC \times PD = Dal = DAL.$

84. $RPr = nn = TP\varrho.$

85. $RTr = \left(\frac{nn}{mm} \times tt = \frac{tt - xx}{xx} \times tt = \right) ATa.$

86. $RTf = \left(\frac{tnz}{mC} \times \frac{fv}{m} = \frac{ft}{m} \times \frac{nC}{m} = \right) CTP = r\,TF.$

87. $NTn = \left(\frac{ffcc}{mm\,nn} \times \overline{ff \backsim mm} = \right) \frac{ffcc}{mm} = PT\,f.$

88. $PN \times CB = \left(\overline{t \backsim x} = \frac{fc}{tn} C = \right) AN \times CQ.$

89. $CAV = \left(t \times \frac{ty}{x} = \frac{tt}{x} \times y = \right) CT \times PD.$

90. $PRh = \left(\frac{cczz}{cc} = \right) \overline{FR}^2.$

Hence Fh is perpendicular to FP.

91. $FMf = \left(\frac{fz}{t} \times \frac{fv}{t} = \right) \frac{ff}{tt} \times zv = \frac{ff}{tt} \times cc = \frac{ff}{tt} \times \overline{CQ}^2$

92. $AMa = \left(tt \backsim \frac{ffmm}{tt} = \right) cc \times \frac{ff}{tt} \times zv$

$= \overline{CB}^2 + \frac{ff}{tt} \times \overline{CQ}^2 = \overline{CB}^2 + FMf.$

93. $\frac{\overline{PN}^2}{TN^2}=\frac{xx}{tt}=\frac{\overline{CD}^2}{AC^2}=\frac{\overline{AN}^2}{VN^2}=\frac{\overline{CA}^2}{CT^2}=\frac{ADa}{ATa}=\frac{ADa}{RTr}=\frac{\overline{AD}^2}{AT^2}=\frac{\overline{aD}^2}{aT^2}=\frac{CDT}{ATa}$.

94. $\frac{FT}{CT}=\frac{z}{t}=\frac{FP}{CA}=\frac{FR}{C\varrho}$. Sim. $\triangle$s TFR, TCϱ.
And $\frac{AD}{DT}=\frac{x}{t+x}=\frac{CD}{aD}=\frac{AC}{aT}$.

95. $\frac{fT}{CT}=\frac{v}{t}=\frac{fP}{AC}=\frac{fr}{C\varrho}$. And $\frac{AT}{DT}=\frac{t}{t+x}=\frac{AC}{aD}=\frac{CT}{aT}$.

96. $\frac{PM}{Pm}=\frac{cc}{tt}=\frac{p}{t}=\frac{\overline{BC}^2}{AC^2}$. And $\frac{Pm}{fm}=\frac{t}{f}=\frac{AC}{CF}$.

97. $\frac{FMf}{PM^2}=\frac{ff}{cc}=\frac{\overline{t+c}\times\overline{t-c}}{cc}=\left(\frac{tt-tp}{tp}=\right)\frac{t-p}{p}=\frac{\overline{CF}^2}{GB^2}$.

98. Let PM, the perpendicular to the tangent PT, cut the axis Aa in M; and fP, produced, cut FR in φ.
Then will PM bisect the angle FPf.
For PF $\times$ Mf = Pf $\times$ MF.

99. And the angle FPT is equal to the angle fPt.
For $\angle$TPF + $\angle$FPM = $\angle t$Pf + $\angle f$PM.

100. Therefore PT will bisect the angle FPφ.
For $\angle\varphi$PT = ($\angle f$Pt =) $\angle$FPT.

101. Consequently Pφ = PF. And Rφ = RF.

102. A circumference of a circle described from t, with the radius tN, will cut the axis Aa in the focii F, f.
For TN $\times$ Tn = TF $\times$ Tf.

103. A

103. A circumference of a circle defcribed from C, with the radius CA, will cut the tangent PT in R, r.

Whence the perpendiculars RF, rf, to that tangent, will cut the axis Aa in the focii F, f.

For $TR \times Tr = TA \times Ta$.

104. A circumference defcribed from B, with AC, in the Ellipfis, or from C, with AB in the Hyperbola, will cut the axis Aa, in the focii F, f.

For, in the Ellipfis, $tt = cc + ff$, or $\overline{AC}^2 = (\overline{BC}^2 + \overline{CF}^2 =) \overline{BF}^2$.

And in the Hyperbola, $ff = tt + cc$, or $\overline{CF}^2 = (\overline{AC}^2 + \overline{BC}^2 =) \overline{AB}^2$.

105. Let CQ produced, cut PF, Pf, in z, x; draw mz, mx, and MZ, MX, parallel to them, cutting PF, Pf in Z, X.

Then $Px = CR = CA = Pz = t$.

106. Hence $\angle Pxz = \angle Pzx$; and $\angle mxz = \angle mzx$.

For Pm is perpendicuiar to zx.

Confequently, the angles Pzm, Pxm; PZM, PXM are equal.

107. And the triangles Pzm, Pxm, are fimilar and equal:

And fo are the triangles PZM, PXM.

Confequently, the trapezias Pzmx, PZMX, are fimilar.

108. Let CR, Cr, cut PF, Pf, in K, k.

Then $CK = \left(\frac{CF \times Pf}{Ff} =\right) \frac{1}{2} Pf = Pk = kr$.

And $Ck = \left(\frac{Cf \times PF}{Ff} =\right) \frac{1}{2} PF = PK = KR$.

109. Alfo $PX = PZ = \left(\frac{PM}{Pm} \times Pz = \frac{cc}{t} \times \frac{c}{tc} \times t =\right) \frac{cc}{t} = \left(\frac{pt}{t} =\right) p$.

110. The Trapezias FGbR, PDFR, frtC, are similar, and consequently their corresponding parts are proportional.

That is, $\left\{\begin{array}{c}\frac{FG}{PD}=\frac{Gb}{DF}=\frac{bR}{FR}=\frac{RF}{RP}\\ \overline{fr}=\overline{rt}=\overline{tC}=\overline{Cf}\end{array}\right\}$

For the triangles RFb, RPF, and FbG, FPD, are similar.

111. The Trapezias CϱPD, tCMP are similar, and consequently their corresponding parts are proportional.

That is, $\frac{C\varrho}{tC}=\frac{\varrho P}{CM}=\frac{PD}{PM}=\frac{DC}{Pt}$.

112. And CR, Cr, are parallel to Pf, PF, and equal to CA.

For RTf = CTP = rTF.

113. Let Σ' = sine of the $\angle tPf$, or TPF; R = tabular radius.

Then $\frac{R}{\Sigma'}=\left(\frac{Pf}{fr}=\right)\frac{C}{c}=\frac{\sqrt{zv}}{c}$.

Put Σ = sine of the $\angle$PCQ, made by any diameter and its ordinate.

Then $\frac{TT}{CC}=\frac{1}{2}ff\pm\sqrt{\frac{1}{4}f^4\backsim\frac{r^2t^2c^2}{\Sigma^2}}$; $\frac{tt}{cc}=\frac{1}{2}FF\pm\sqrt{\frac{1}{4}F^4\backsim\frac{T^2C^2\Sigma^2}{rr}}$.

114. Let the parallels PFΠ, Cβr, $pf\pi$ be drawn, cutting the curve in Π, β, π; and ordinately applied to some diameter (2τ), whose parameter is 2ϖ, and semiconjugate Cβ = χ, to which Pπ is ordinately applied at δ.

Then $C\delta=\left(\frac{PF+\pi f}{2}=\frac{PF+F\Pi}{2}=\right)\frac{1}{2}z+\frac{1}{2}z'=\frac{1}{2}P\Pi=\left(\frac{\overline{C\beta}^2}{Cr}=\right)\frac{\overline{C\beta}^2}{AC}=\frac{\chi\chi}{t}=\frac{\varpi\tau}{t}$.

115. $\overline{C\beta}^2=\left(\frac{1}{2}AC\times P\Pi=\right)\frac{1}{2}t\times\overline{z+z'}=\left(\overline{AC}^2\times\frac{PF\Pi}{APa}=\right)\frac{tt}{cc}zz'=\frac{t}{p}zz'$.

116. $\frac{FP}{F\Pi}=\frac{z}{z'}=\frac{RD}{F\Delta}=\frac{tz-cc}{cc-tz'}=\frac{z-p}{p-z'}$.

117. $PF\Pi=\frac{pzz}{2z-p}=\frac{1}{2}p\times\overline{z+z'}=\frac{1}{2}p\times P\Pi$.

118. $P\Pi=z+z'=\frac{2zz}{2z-p}=\frac{2\varpi\tau}{t}=\frac{\tau}{t}\times 2\varpi$.

$$\overline{CB}^2 = cc = pt = \pm tt \mp ff = \pm \overline{t \mp f} \times \overline{t+f} = AFa = VAU = tCd = CFG$$
$$= PM \times C\varrho = FR \times fr = AN \times an = al \times AL = AC \times F\Phi = tt \times \frac{PM}{Pm}$$
$$= tt \times \frac{DM}{DC} = tt \times \frac{dC}{dm} = tt \times \frac{\overline{PM}^2}{\overline{CQ}^2} = tt \times \frac{\overline{PD}^2}{ADa} = tt \times \frac{\overline{PD}^2}{TDC} = ff \times \frac{PD}{Cm}$$
$$= \overline{\frac{CQ \times C\varrho}{AC}}\Big|^2 = \overline{\frac{AN \times CQ}{PN}}\Big|^2 = AMa - FMf = FPf - RPr = ff \times \frac{t}{t-p}$$
$$= TT + CC - tt = \frac{tt}{xx} \times \overline{zv + xx - tt} = \tfrac{1}{2} zv \pm \sqrt{\tfrac{1}{4} z^2 v^2 ffyy}.$$

$$\overline{CQ}^2 = CC = P \times T = MPm = NPn = TPt = FPf = CPn = Lpl$$
$$= PN \times pL = CD \times TM = PD \times tm = \frac{tt}{cc} \times \overline{PM}^2 = \frac{cc}{tt} \times \overline{Pm}^2 = \frac{cc}{ff} \times \overline{fm}^2$$
$$= \frac{tt}{ff} \times FMf = \overline{\frac{ct}{ff} \times Mm}\Big|^2 = \overline{\frac{ACB}{C\varrho}}\Big|^2 = \overline{PM}^2 + FMf = \overline{BC}^2 + RPr$$
$$= tt + cc - TT = tt - \frac{ff}{tt} xx = \frac{tt}{cc} yy + \frac{cc}{tt} xx = \frac{DC}{DT} \times \overline{PT}^2.$$

$$\overline{CF}^2 = ff = tt - cc = tt - tp = \frac{cc}{p} \times \overline{t-p} = \frac{tt}{xx} \times mm = P\varrho \times Tt$$
$$= \frac{tt}{CC} \times FMf = \frac{cc}{CC} \times \overline{fm}^2 = \frac{mm}{CC} \times FTf = \frac{mm}{CC} \times NTn.$$

The semi-parameter (p) to the greater axis (Aa) is equal to

$$F\Phi = PZ = \frac{AFa}{AC} = \frac{VaU}{AC} = \frac{CFG}{AC} = \frac{tCd}{AC} = \frac{PF\Pi}{\frac{1}{2}P\Pi} = \frac{PM \times C\varrho}{AC}$$
$$= \frac{FR \times fr}{AC} = \frac{AN \times an}{AC} = \frac{AC \times PM}{Pm} = \frac{AC \times DM}{DC} = \frac{AC \times dC}{dm} = \frac{BC \times PM}{CQ}$$
$$= \frac{BC \times CQ}{Pm} = \frac{\overline{BC}^2}{AC} = \frac{cc}{t} = \frac{ttyy}{sx} = \frac{\pm tt \mp ff}{t} = \frac{cc - yy}{xx} \times t$$
$$= \frac{CC + TT - tt}{t} = \frac{cc + xx - tt}{xx} \times t = \frac{2t - z}{tz} \times \overline{FR}^2 = \frac{2t - v}{tv} \times \overline{fr}^2.$$

The semi-parameter (P) to any diameter (Pp) is equal to

$$Pn = \frac{\overline{CQ}^2}{PC} = \frac{CC}{T} = \frac{MPm}{PC} = \frac{NPn}{PC} = \frac{TPt}{PC} = \frac{FPf}{PC} = \frac{Lpl}{PC} = \frac{PN \times pL}{PC}$$
$$= \frac{CD \times TM}{PC} = \frac{PD \times tm}{PC} = \frac{PT \times PN}{PV} = \frac{tt}{cc} \times \frac{\overline{PM}^2}{PC} = \frac{cc}{tt} \times \frac{Pm^2}{PC}$$
$$= \frac{cc}{ff} \times \frac{\overline{fm}^2}{PC} = \frac{tt}{ff} \times \frac{PMf}{PC} = \frac{tt + cc - TT}{T}.$$

[See TAB. XIV, XV.]

XXXVII. *An*

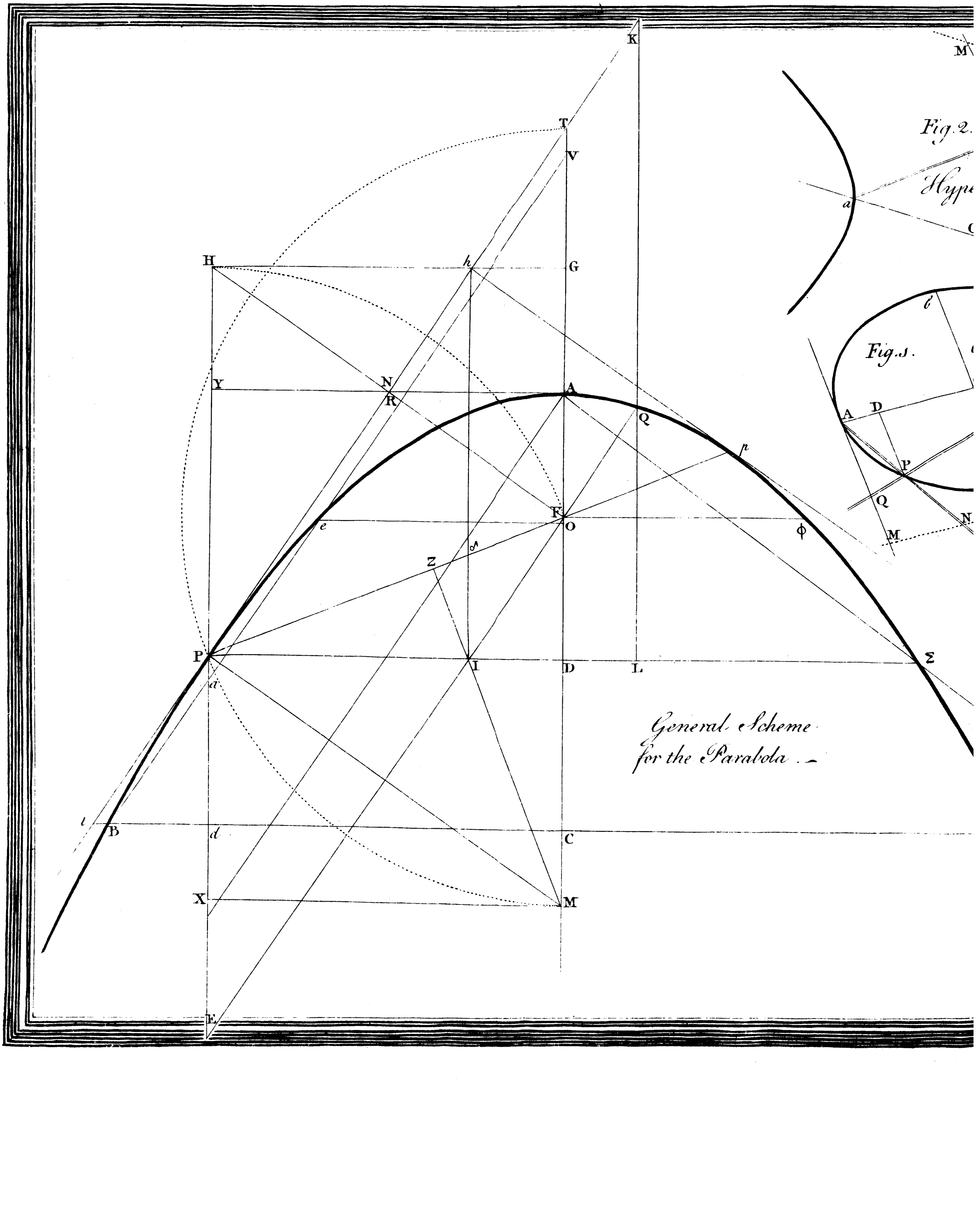


Fig. 2.
Fig. 5.
General Scheme
for the Parabola.

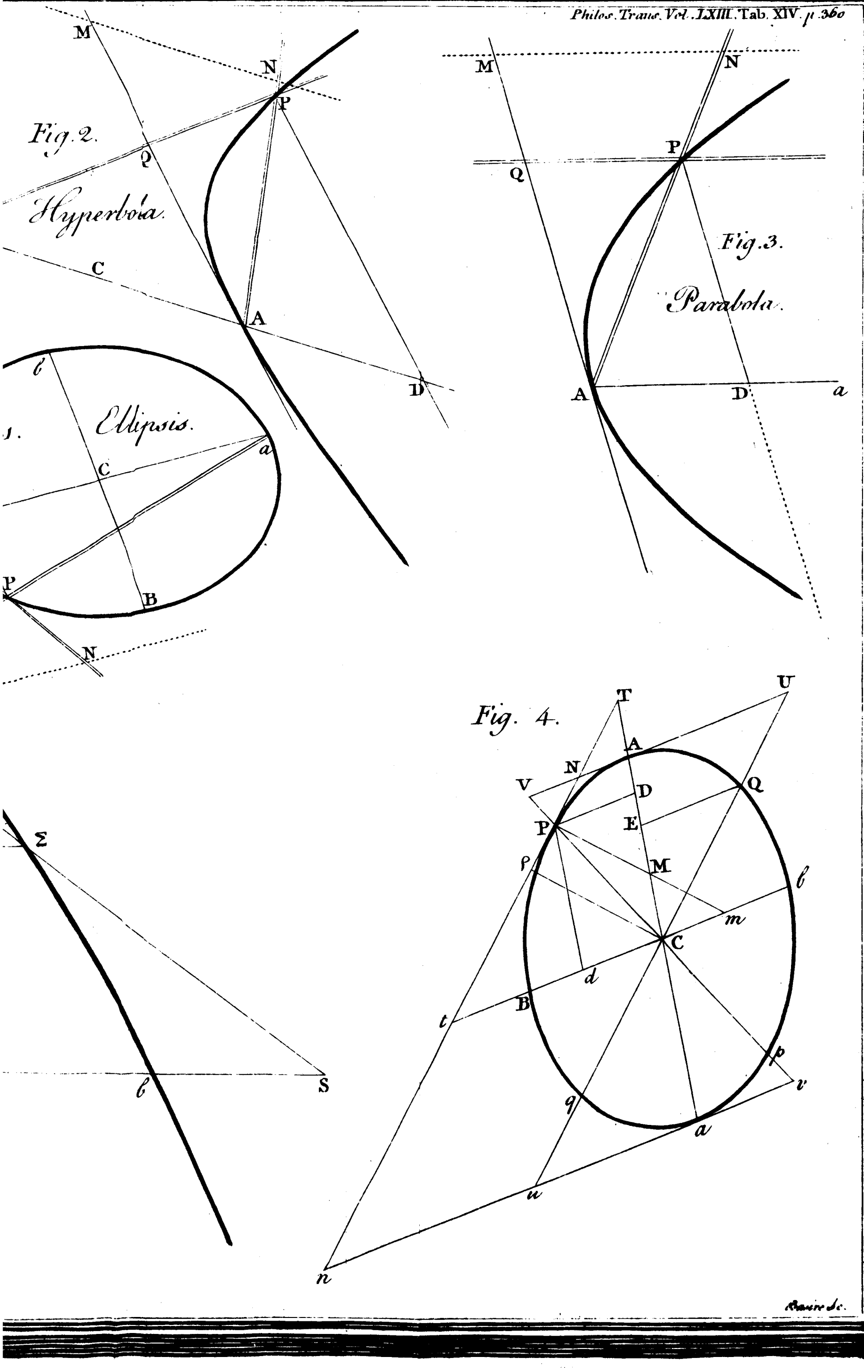
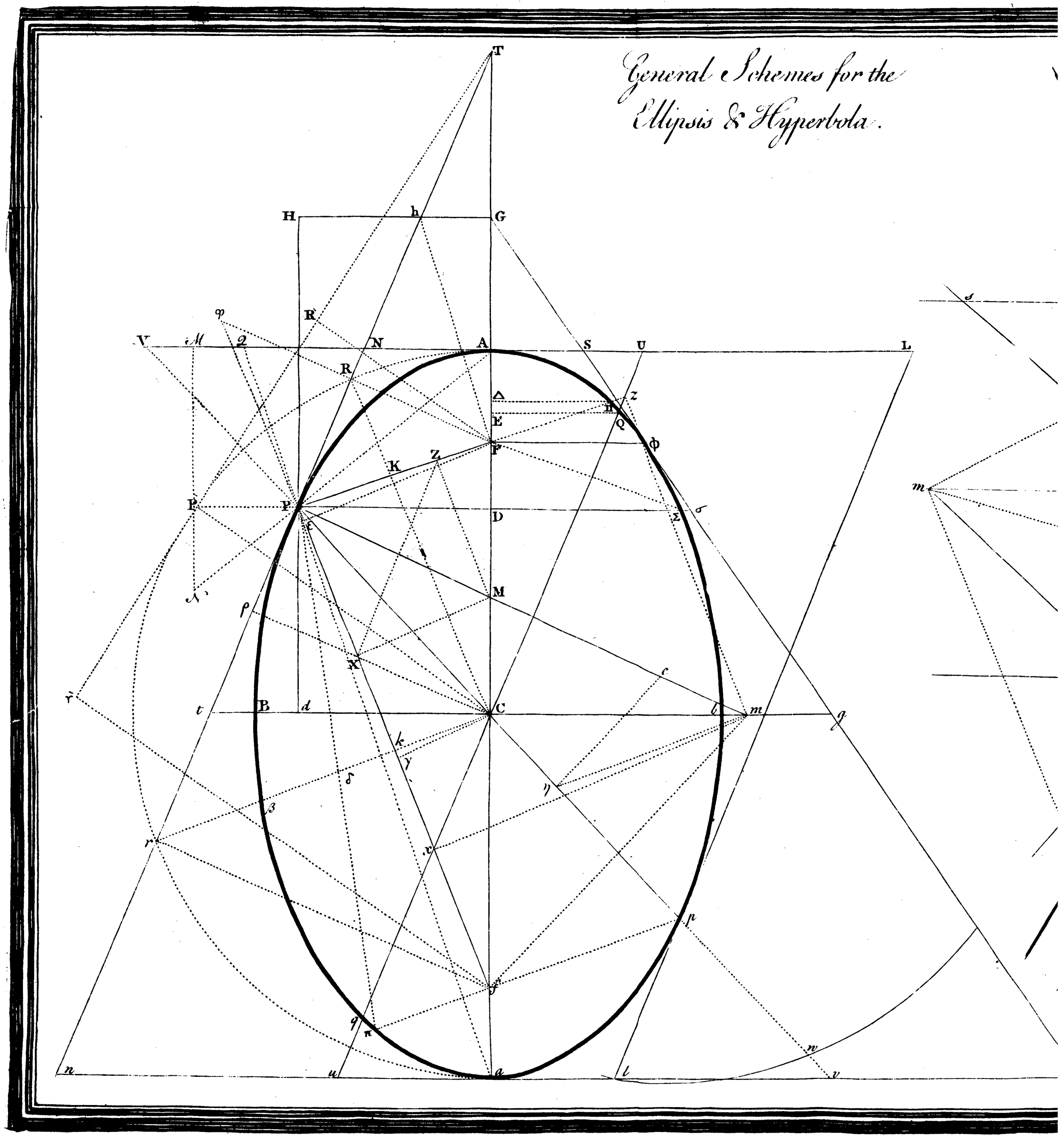
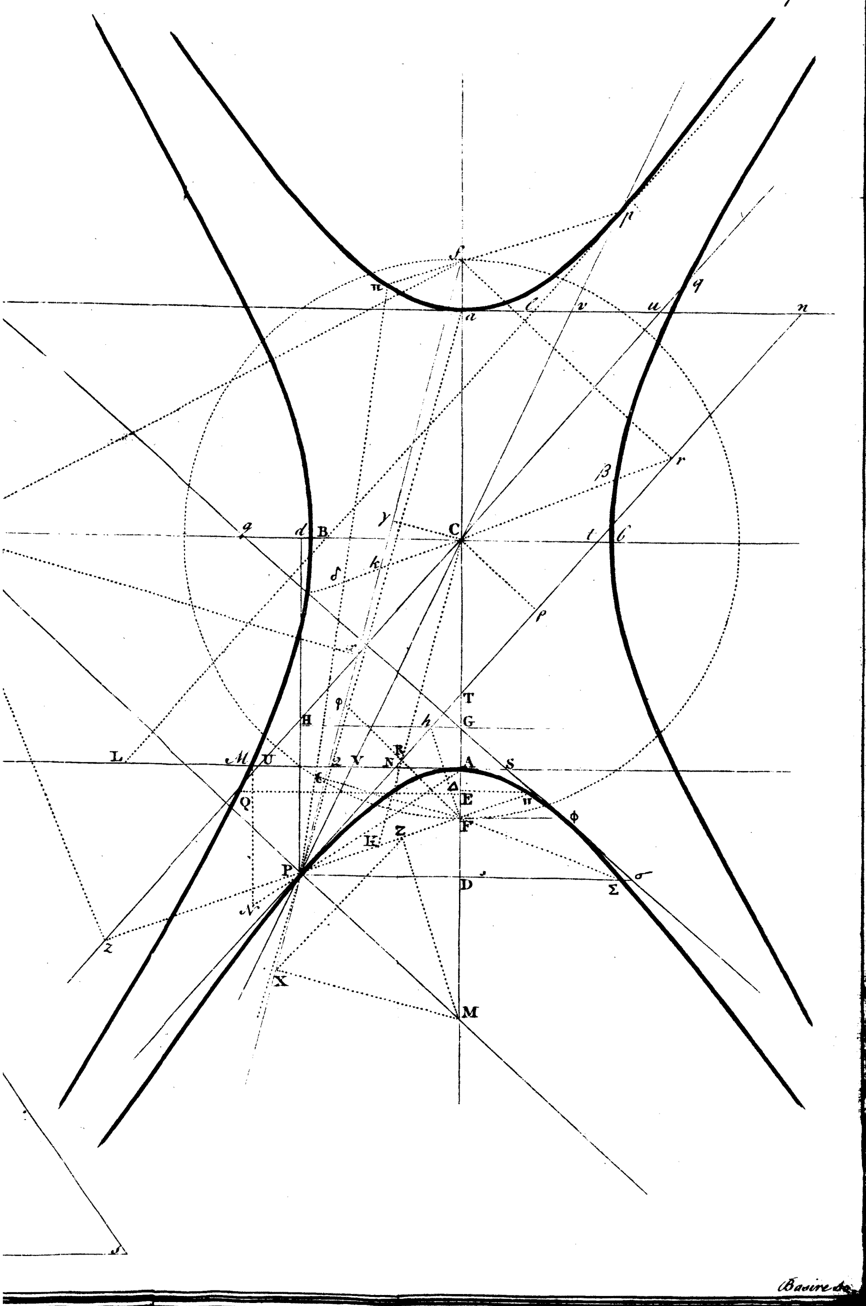
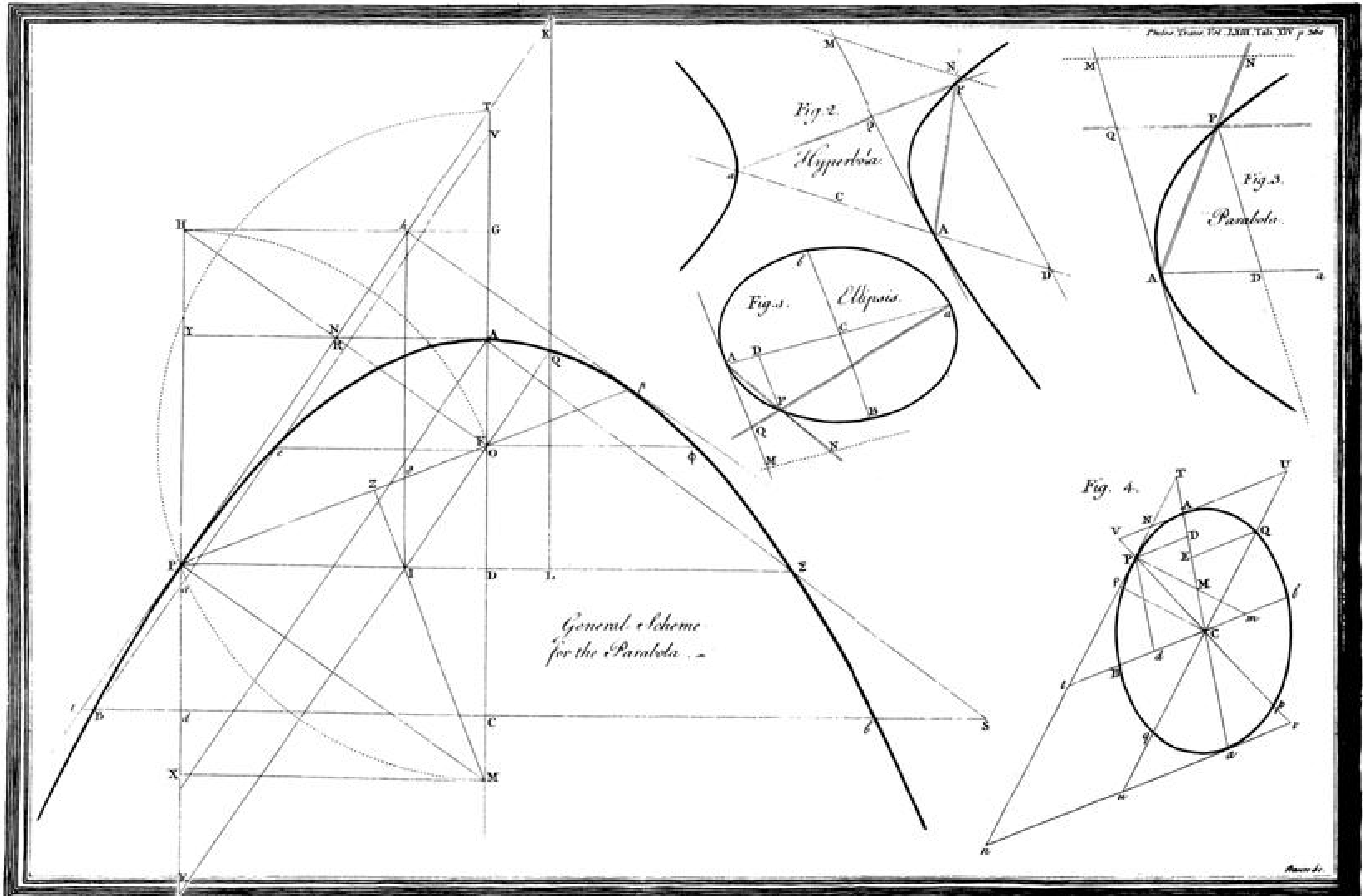


Fig. 2.
Hyperbola.
Fig. 3.
Parabola.
Ellipsis.
Fig. 4.
Basire sc.

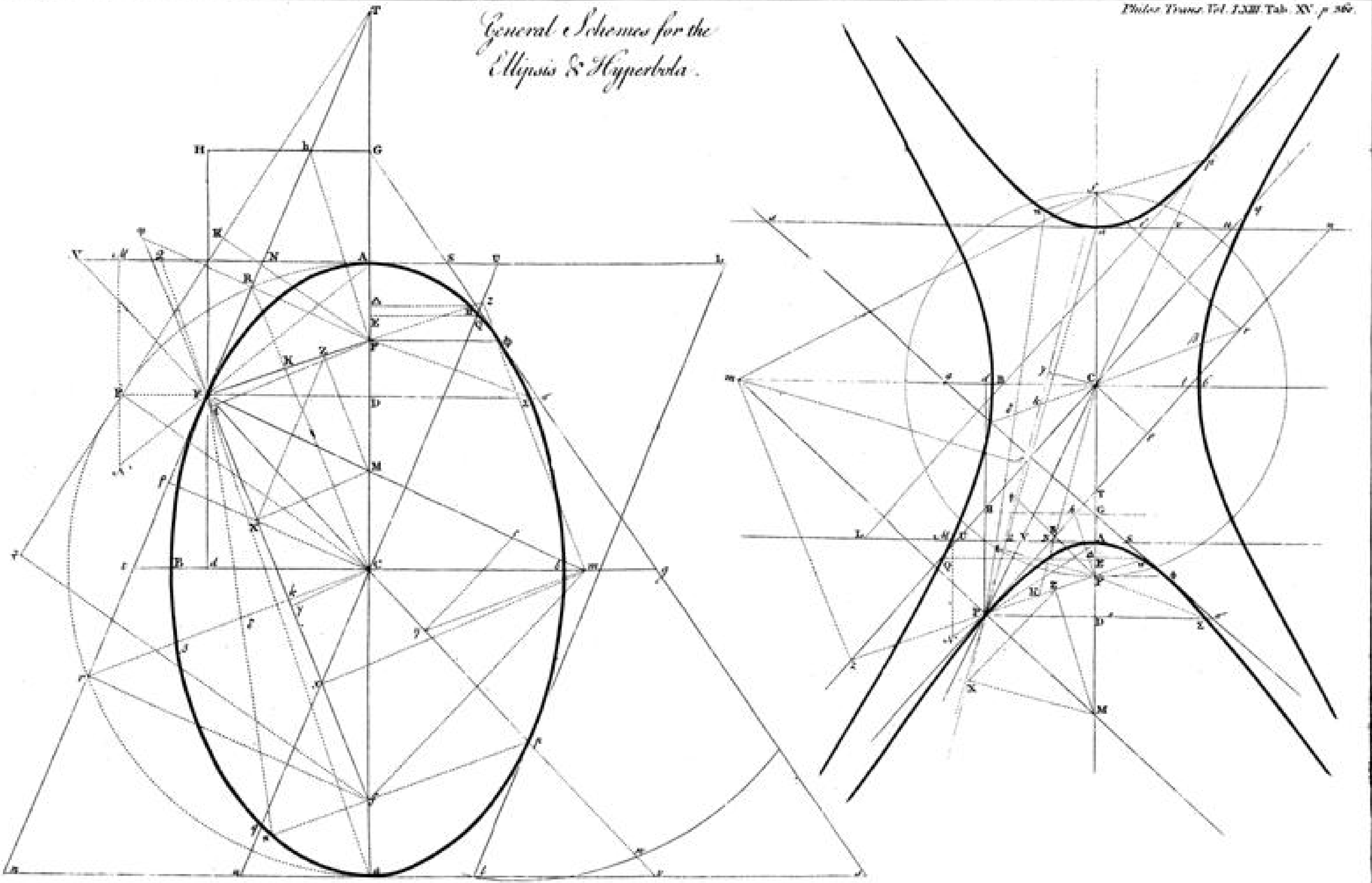


General Schemes for the Ellipsis & Hyperbola.


Basire sc.


Philos. Trans. Vol. LXIII. Tab. XIV.
Fig. 2.
Hyperbola
Fig. 1.
Ellipsis
Fig. 3.
Parabola
Fig. 4.
General Scheme
for the Parabola

General Schemes for the Ellipsis & Hyperbola.



Basire sc.